

Section 1.4: Solving Absolute Value Equations

absolute value equation: an equation in which the variable is contained within the absolute value

extraneous solution: a solution that does not satisfy the original equation

Solving Absolute Value Equations

To solve $|ax + b| = c$ when $c \geq 0$, solve the related linear equations

$$ax + b = c \quad \text{or} \quad ax + b = -c.$$

When $c < 0$, the absolute value equation $|ax + b| = c$ has no solution because absolute value always indicates a number is not negative.

STEPS TO SOLVE AN ABSOLUTE VALUE EQUATION

1. Isolate the absolute value expression
2. Create 2 equations
3. Solve both equations
4. Check your solution(s).

Solve the equation. Graph the solution(s), if possible.

1. $|y + 2| = 8$

$$\begin{array}{r|l} y+2=8 & y+2=-8 \\ -2 & -2 \\ \hline y=6 & y=-6 \end{array}$$

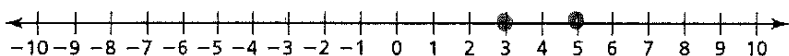
$$\{-6, 6\}$$



2. $|2y - 8| = 2$

$$\begin{array}{r|l} 2y-8=2 & 2y-8=-2 \\ +8 & +8 \\ \hline 2y=10 & 2y=6 \\ \frac{2y}{2} & \frac{2y}{2} \\ y=5 & y=3 \end{array}$$

$$\{3, 5\}$$



Solve the equation. Check for extraneous solutions.

<p>3. $k-5 +4=16$</p> $\begin{array}{r} -4 \quad -4 \\ \hline k-5 =12 \end{array}$ <table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="padding: 5px;">$k-5=12$</td> <td style="padding: 5px;">$k-5=-12$</td> </tr> <tr> <td style="padding: 5px;">$\begin{array}{r} +5 \quad +5 \\ \hline k=17 \end{array}$</td> <td style="padding: 5px;">$\begin{array}{r} +5 \quad +5 \\ \hline k=-7 \end{array}$</td> </tr> </table> <p style="text-align: center; border: 1px solid black; border-radius: 10px; padding: 5px; width: fit-content; margin: 10px auto;">$\{-7, 17\}$</p>	$k-5=12$	$k-5=-12$	$\begin{array}{r} +5 \quad +5 \\ \hline k=17 \end{array}$	$\begin{array}{r} +5 \quad +5 \\ \hline k=-7 \end{array}$	<p>4. $2\left \frac{d}{3}\right =6$</p> $\begin{array}{r} \frac{d}{3} \\ \hline \end{array}$ $\left \frac{d}{3}\right =3$ <table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="padding: 5px;">$\frac{d}{3}=3$</td> <td style="padding: 5px;">$\frac{d}{3}=-3$</td> </tr> <tr> <td style="padding: 5px;">$d=9$</td> <td style="padding: 5px;">$d=-9$</td> </tr> </table> <p style="text-align: center; border: 1px solid black; border-radius: 10px; padding: 5px; width: fit-content; margin: 10px auto;">$\{-9, 9\}$</p>	$\frac{d}{3}=3$	$\frac{d}{3}=-3$	$d=9$	$d=-9$
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<p>5. $3 x+5 +10=28$</p> $\begin{array}{r} -10 \quad -10 \\ \hline 3 x+5 =18 \\ \frac{3}{3} \quad \frac{18}{3} \\ x+5 =6 \end{array}$ <table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="padding: 5px;">$x+5=6$</td> <td style="padding: 5px;">$x+5=-6$</td> </tr> <tr> <td style="padding: 5px;">$\begin{array}{r} -5 \quad -5 \\ \hline x=1 \end{array}$</td> <td style="padding: 5px;">$\begin{array}{r} -5 \quad -5 \\ \hline x=-11 \end{array}$</td> </tr> </table> <p style="text-align: center; border: 1px solid black; border-radius: 10px; padding: 5px; width: fit-content; margin: 10px auto;">$\{-11, 1\}$</p>	$x+5=6$	$x+5=-6$	$\begin{array}{r} -5 \quad -5 \\ \hline x=1 \end{array}$	$\begin{array}{r} -5 \quad -5 \\ \hline x=-11 \end{array}$	<p>6. $x-7 =-5$</p> <p style="text-align: center; border: 1px solid black; border-radius: 10px; padding: 5px; width: fit-content; margin: 10px auto;">\uparrow No Solution</p>				
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<p>7. $2x-1 +4=4$</p> $\begin{array}{r} -4 \quad -4 \\ \hline 2x-1 =0 \end{array}$ <table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="padding: 5px;">$2x-1=0$</td> <td rowspan="3" style="padding: 5px; vertical-align: middle;">No need for a second equation because 0 cannot be negative.</td> </tr> <tr> <td style="padding: 5px;">$\begin{array}{r} +1 \quad +1 \\ \hline 2x=1 \\ \frac{2}{2} \quad \frac{1}{2} \\ x=\frac{1}{2} \end{array}$</td> </tr> </table> <p style="text-align: center; border: 1px solid black; border-radius: 10px; padding: 5px; width: fit-content; margin: 10px auto;">$\left\{\frac{1}{2}\right\}$</p>	$2x-1=0$	No need for a second equation because 0 cannot be negative.	$\begin{array}{r} +1 \quad +1 \\ \hline 2x=1 \\ \frac{2}{2} \quad \frac{1}{2} \\ x=\frac{1}{2} \end{array}$	<p>8. $5x -10=-5$</p> $\begin{array}{r} +10 \quad +10 \\ \hline 5x =5 \end{array}$ <table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="padding: 5px;">$\frac{5x}{5}=\frac{5}{5}$</td> <td style="padding: 5px;">$\frac{5x}{5}=\frac{-5}{5}$</td> </tr> <tr> <td style="padding: 5px;">$x=1$</td> <td style="padding: 5px;">$x=-1$</td> </tr> </table> <p style="text-align: center; border: 1px solid black; border-radius: 10px; padding: 5px; width: fit-content; margin: 10px auto;">$\{-1, 1\}$</p>	$\frac{5x}{5}=\frac{5}{5}$	$\frac{5x}{5}=\frac{-5}{5}$	$x=1$	$x=-1$	
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